ДИС Кирчев 25.11.12

Правило на Лопитал

$\lim_{x \to a} f(x) = 0$ $\lim_{x \to a} g(x) = \infty$

$x \in (a,b)$ $a < x < b$

$\lim \frac{f(x)}{g(x)}$ искаме $f(x)$ и $g(x)$ да са диференцируеми в интервала (a,b)

искаме $g'(x) \neq 0$ и да е непрекъсната

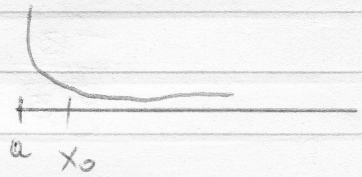
$$\boxed{\lim \frac{f'(x)}{g'(x)} = e} \quad \Phi$$

Док-во

1) при $e < \infty$ (крайна) $\exists \varepsilon > 0$

избираме такова x_0 близо до a т.ге

$$\boxed{\left| \frac{f'(x)}{g'(x)} - e \right| < \frac{\varepsilon}{3}} \quad \Phi$$



$$\left| \frac{f(x)}{g(x)} - e \right| = \left| \frac{f(x)}{g(x)} - \frac{f(x_0) - f(x)}{g(x_0) - g(x)} + \frac{f(x_0) + f(x)}{g(x_0) - g(x)} - e \right| \leq$$

$$\left| \frac{f(x)}{g(x)} - \frac{f(x_0) - f(x)}{g(x_0) - g(x)} \right| + \left| \frac{f(x_0) + f(x)}{g(x_0) - g(x)} - e \right| \Rightarrow$$

$$\frac{f'(3)}{g'(3)} - e < \frac{\varepsilon}{3}$$

$$\left| \frac{f(x_0)-f(x)}{g(x_0)-g(x)} \left(\frac{f(x)}{g(x)} \frac{g(x)-g(x_0)}{f(x)-f(x_0)} - 1 \right) \right| =$$

$$\left| \frac{f(x_0)-f(x)}{g(x_0)-g(x)} \left(\frac{g(x)-g(x_0)}{g(x)} + \frac{f(x)}{g(x)} \frac{g(x)-g(x_0)}{f(x)-f(x_0)} - 1 \right) \right| =$$

$$\left| \frac{f(x_0)-f(x)}{g(x_0)-g(x)} \right| \underbrace{\left(\frac{g(x)-g(x_0)}{g(x)} - 1 \right)}_{-\frac{g(x_0)}{g(x)}} + \frac{f(x)}{g(x)} \Rightarrow$$

$$\underbrace{\left| \frac{f(x_0)-f(x)}{g(x_0)-g(x)} \right|}_{\frac{f'(3)}{g'(3)} < C} - \underbrace{\frac{g(x_0)}{g(x)}}_{0} + \underbrace{\frac{f(x)}{g(x)}}_{0} \Rightarrow$$

$$\left| \frac{f(x)}{g(x)} - \ell \right| \leq \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$$